

① Handout & SmartBoard about
MVT - website examples

② Find value of c

$$\text{Let } f(x) = x^2 \text{ on } [0, 3]$$

① is $f(x)$ continuous on $[a, b]$?

② is $f(x)$ differentiable on (a, b) ?

③ Find value of c .

$$A_{\text{Rec}} = \frac{f(3) - f(0)}{3 - 0} \quad f'(x) = 2x$$

$$= \frac{9 - 0}{3}$$

$$= 3$$

$$\text{Set } f'(c) = A_{\text{Rec}}$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Practice Pg 202 #1-5, 9-11

Pa 202 #1-5, 9-11

① $f(x) = x^2 + 2x - 1$, $[0, 1]$

$$\Delta_{\text{Roc}} = \frac{f(1) - f(0)}{1 - 0} \quad f'(x) = 2x + 2$$

$$= \frac{2 - (-1)}{1}$$

$$= 3$$

$$2c + 2 = 3$$

$$c = \frac{1}{2}$$

② $f(x) = x^{2/3}$, $[0, 1]$

$$\Delta_{\text{Roc}} = \frac{f(1) - f(0)}{1 - 0} = 1, \quad f'(x) = \frac{2}{3} x^{-1/3}$$

$$\frac{2}{3\sqrt[3]{c}} = 1$$

$$c = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

③ $f(x) = x^{1/3}$, $[-1, 1]$

Not differentiable

at $x = 0$

(vertical tangent)

④ $f(x) = |x - 1|$, $[0, 1]$

Not differentiable

at $x = 1$, left $\neq$ right

⑤ $f(x) = \sin^{-1}(x)$, $[-1, 1]$

$$\Delta_{\text{Roc}} = \frac{f(1) - f(-1)}{1 - (-1)}, \quad f'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{\frac{\pi}{2} - (-\frac{\pi}{2})}{2}$$

$$= \frac{\pi}{2}$$

$$\frac{1}{\sqrt{1-c^2}} = \frac{\pi}{2}$$

$$\frac{4}{\pi} = 1 - c^2$$

$$c = \sqrt{1 - \frac{4}{\pi^2}}$$

⑥ $f(x) = x + \frac{1}{x}$, $\frac{1}{2} \leq x \leq 2$

a)

$$\Delta_{\text{Roc}} = \frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}}$$

$$= \frac{(2 + \frac{1}{2}) - (\frac{1}{2} + 2)}{\frac{3}{2}}$$

$$= 0$$

$$\text{Point } (2, f(2)) = (2, \frac{5}{2}) \quad y = \frac{5}{2}$$

b) $f'(x) = 1 - \frac{1}{x^2}$

$$1 - \frac{1}{c^2} = 0$$

$$c = 1$$

$$\text{Point } (1, f(1)) = (1, 2) \quad y = 2$$

10) $f(x) = \sqrt{x-1}$, $1 \leq x \leq 3$

a) ARoc: $\frac{f(3) - f(1)}{3 - 1}$

$= \frac{\sqrt{2}}{2}$

Point $(1, f(1)) = (1, 0)$

$y = \frac{\sqrt{2}}{2}(x - 1)$

b) $f'(x) = \frac{1}{2\sqrt{x-1}}$

$\frac{\sqrt{2}}{2} = \frac{1}{2\sqrt{c-1}}$

$c - 1 = \frac{1}{2}$

$c = \frac{3}{2}$

Point: $(\frac{3}{2}, f(\frac{3}{2})) = (\frac{3}{2}, \frac{1}{\sqrt{2}})$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(x - \frac{3}{2})$

11) Ave speed = $\frac{159}{2} = 79.5$

MVT says at least
once trucker was
going 79.5 mph

